# Cambridge International AS & A Level

CANDIDATE NAME					
CENTRE NUMBER			CANDIDATE NUMBER		

# 5202311540

#### **FURTHER MATHEMATICS**

9231/13

Paper 1 Further Pure Mathematics 1

May/June 2021

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

#### **INSTRUCTIONS**

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do not write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

#### **INFORMATION**

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has 20 pages. Any blank pages are indicated.

1 (a) Show that

	$\tan(r+1) - \tan r = \frac{\sin 1}{\cos(r+1)\cos r}.$	[2]
<b>b</b> )	Use the method of differences to find $\sum_{r=1}^{n} u_r$ .	[3

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**(c)** 

(a)	Stat	te the value of $S_1$ and find the value of $S_2$ .	
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(b)	<b>(3)</b>	Everyone C in towns of C and C	
(D)	(1)	Express $S_{n+3}$ in terms of $S_{n+2}$ and $S_n$ .	
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	<b>(**</b> )		
	(ii)	Hence, or otherwise, find the value of $S_4$ .	

simplify	an equatio	n whose r	oots are o	$\alpha + \beta, \beta$	$+\gamma, \gamma+$	α.				
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3 (a) Prove by mathematical induction that, for all positive integers n,

$\sum_{r=1}^{n} \left(5r^4 + r^2\right)$	$= \frac{1}{2}n^2(n+1)^2(2n+1).$	[6]

01 1	the result given in part, fully factorising you	ır answer.				7-1	
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4 The matrices **A**, **B** and **C** are given by

$$\mathbf{A} = \begin{pmatrix} 2 & k & k \\ 5 & -1 & 3 \\ 1 & 0 & 1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \text{ and } \quad \mathbf{C} = \begin{pmatrix} 0 & 1 & 1 \\ -1 & 2 & 0 \end{pmatrix},$$

where k is a real constant.

Given that $A$ is singular, find the value of $k$ .	

Jsing the value of $k$ from part (b), find the equations of the invariant li he transformation in the $x$ - $y$ plane represented by <b>CAB</b> .	Į.
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- 5 The curve C has polar equation  $r = \frac{1}{\pi \theta} \frac{1}{\pi}$ , where  $0 \le \theta \le \frac{1}{2}\pi$ .
  - (a) Sketch *C*. [3]

(b) Show that the area of the region bounded by the half-line  $\theta = \frac{1}{2}\pi$  and C is  $\frac{3-4\ln 2}{4\pi}$ . [6]

	e plane $\Pi_1$ contains $l_1$ and the point P with position vector $-2\mathbf{i} - 2\mathbf{j} + 4\mathbf{k}$ .	
(a)	Find an equation of $\Pi_1$ , giving your answer in the form $\mathbf{r} = \mathbf{a} + \lambda \mathbf{b} + \mu \mathbf{c}$ .	[2]
The	e plane $\Pi_2$ contains $l_2$ and is parallel to $l_1$ .	
<b>(b)</b>	Find an equation of $\Pi_2$ , giving your answer in the form $ax + by + cz = d$ .	[4]


Find	the position	vector of th	ne foot of th	e perpendio	cular from	the point $Q$ to $Q$	$\Pi_2$ .	[4
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(a)	Find the equations of the asymptotes of $C$ .	
		•••••
<b>(b)</b>	Find the coordinates of any stationary points on <i>C</i> .	

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[3]

<b>(c)</b>	Sketch <i>C</i> , stating the coordinates of the intersections with the axes.

(d) Sketch the curve with equation  $y = \left| \frac{x^2 - x - 3}{1 + x - x^2} \right|$  and find in exact form the set of values of x for which  $\left| \frac{x^2 - x - 3}{1 + x - x^2} \right| < 3$ . [6]

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## **Additional Page**

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